Question A5/B1 (20 marks) A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in Figure 1. Find the magnitude of the electric field in the regions:

(a) (5 marks) $r < a$ (write your answer in terms of Q, not ρ)

(b) (5 marks) $a < r < b$

(c) (5 marks) $b < r < c$

(d) (5 marks) $r > c$

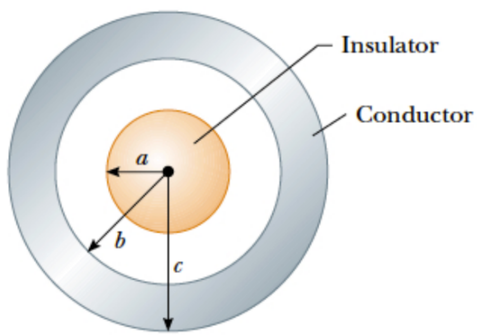


Figure 1: Concentric conducting shell and insulating sphere.

Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ gives

$$EA = \frac{Q_{encl}}{\varepsilon_0}$$

$$E = \frac{1}{4\pi r^2} \frac{Q_{encl}}{\varepsilon_0}$$ for a spherical Gaussian surface

From here, the solution requires figuring out what the enclosed charge, Q_{encl}, is for the given range of radii.

a) $Q_{encl} = \rho V_r = \frac{Q}{V_a} V_r$ $\quad V_r$ - volume enclosed by sphere at r

$$= \frac{Q}{\frac{4}{3}\pi a^3} \cdot \frac{4}{3}\pi r^3$$

$$= Q \frac{r^3}{a^3}$$

This gives $E = \frac{Qr}{4\pi a^3 \varepsilon_0}$

b) $Q_{encl} = Q$, so $E = \frac{Q}{4\pi \varepsilon_0 r^2}$

c) Electric field is zero inside a conductor, so $Q_{encl} = 0$ and $E = 0$.

d) At $r > c$, the charge enclosed is $Q_{encl} = Q$, so the answer is $E = \frac{Q}{4\pi\varepsilon_0 r^2}$.

Question A4/B2 (20 Marks) An initially uncharged 4.50×10^{-6} F capacitor and a 6.00×10^3 Ω resistor are connected in series to a 2.50 V battery that has negligible internal resistance.

(a) (5 marks) What is the initial current of the circuit?

(b) (5 marks) Calculate the time constant for the circuit.

(c) (10 marks) How much time must elapse from the closing of the circuit for the current to decrease to 15% of its initial value?

B2

a) $I_0 = \frac{\mathcal{E}}{R} = \frac{2.50\text{ V}}{6000\ \Omega} = 4.17 \times 10^{-4}$ A

b) $\tau = RC = (6000\ \Omega)(4.50 \times 10^{-6}\text{ F}) = 0.0270$ s

c) $I = I_0 e^{-t/\tau}$

$I/I_0 = 15\% = 0.15$

Rearrange to get $t = -\tau \ln\left(\frac{I}{I_0}\right) = -(0.0270\text{ s})\ln(0.15) = 0.0512$ s

A4

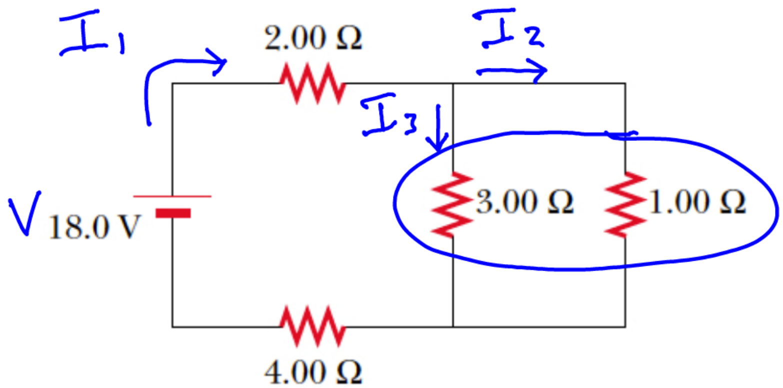
The same except some of the variables were different. The solutions are:

a) 2.14×10^{-4} A

b) 0.0245 s

c) 0.0734 s

Question A1/B3 (20 marks) Calculate the power delivered to the 1.00 Ω and 4.00 Ω resistors in the circuit shown in Figure 2.



$$R_p = \left(\frac{1}{3.00\,\Omega} + \frac{1}{1.00\,\Omega}\right) = 0.750\,\Omega$$

Figure 2: Circuit for Question 3.

With R_p, the total resistance for the circuit can be found:

$$R_{TOT} = (2.00\,\Omega + 0.750\,\Omega + 4.00\,\Omega) = 6.75\,\Omega$$

$$I_1 = \frac{V}{R_{TOT}} = \frac{18.0\text{ V}}{6.75\,\Omega} = 2.67\text{ A}$$

The power delivered to the 2.00 Ω resistor is:

$$P_2 = I_1^2 R = (2.67\text{ A})^2(2.00\,\Omega) = \underline{14.2\text{ W}}$$

Similarly, $P_4 = I_1^2 R = (2.67\text{ A})^2(4.00\,\Omega) = \underline{28.4\text{ W}}$

Now we need I_2 & I_3 to get P_1 and P_3.

$$I_1 = I_2 + I_3 = 2.67\text{ A}$$
$$I_2 = 2.67\text{ A} - I_3$$

Using $\Sigma V = 0$ around a loop: $-1I_2 + 3I_3 = 0$

$$-(2.67 - I_3) + 3I_3 = 0$$
$$I_3 = 0.668\text{ A}$$

$$P_3 = I_3^2 R = (0.668\ \text{A})^2 (3.00\ \Omega) = \underline{1.34\ \text{W}}$$

$$I_2 = 3 I_3$$

$$I_2 = 2.00\ \text{A}$$

$$P_1 = I_2^2 R = (2.00\ \text{A})^2 (1.00\ \Omega) = \underline{4.00\ \text{W}}$$

Question A2/B4 (20 marks) Two protons are released from rest when they are 0.550×10^{-9} m apart.

(a) (10 marks) When do they reach their maximum speed? Calculate the maximum speed.

(b) (10 marks) The force between the protons is given by

$$F = \frac{U}{r} \tag{1}$$

where U is the potential energy function and r is the distance between the protons. When do the protons achieve their maximum acceleration? Calculate the maximum acceleration.

For this problem, see solution to DG07, question 3.

Question A3/B5 (20 Marks) Figure 3 shows a system of five capacitors, where the potential difference across ab is 20.0 V. (Note: 1 nF $= 1 \times 10^{-9}$ F)

(a) (10 marks) Find the equivalent capacitance of this system between a and b.

(b) (10 marks) How much charge is stored by this combination of capacitors?

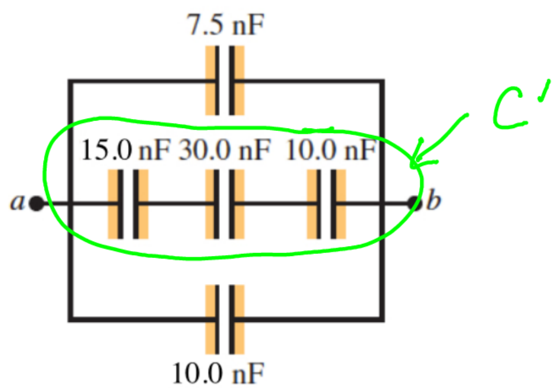


Figure 3: Five capacitor arrangement.

B5| a) $\frac{1}{C'} = \left(\frac{1}{15.0} + \frac{1}{30.0} + \frac{1}{10.0}\right) nF^{-1}$

$C' = 5.0\ nF$

C_{eq} for whole combination

$C_{eq} = 7.5\ nF + 5.0\ nF + 10.0\ nF = 22.5\ nF$

b) $Q = C_{eq} V_{ab} = (22.5 \times 10^{-9}\ F)(20.0\ V) = 4.5 \times 10^{-7}\ C$

A3| One of the capacitors and V_{ab} were different. The solutions are : a) $C' = 5.0\ nF$

b) $Q = (19.0 \times 10^{-9}\ F)(25.0\ V) = 4.75 \times 10^{-7} C$